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Interface resistances and AC transport in a Luttinger liquid

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Abstract. We consider a Luttinger liquid (LL) connected to two reservoirs when the two sample-reservoir interface resistances R_S and R_D are arbitrary (not necessarily quantized at half-the-quantum of resistance). We compute exactly the dynamical impedance of a Luttinger liquid and generalize earlier expressions for its dynamical conductance in the following situations. (i) We first consider a gated Luttinger liquid. It is shown that the Luttinger liquid parameters u and K and the interface resistances R_S and R_D can be experimentally determined by measuring both the dynamical conductance and impedance of a gated wire at second order in frequency. The parallel law addition for the charge relaxation resistance R_q is explicitly recovered for these non-trivial interface resistances as $R_q^{-1} = R_S^{-1} + R_D^{-1}$. (ii) We discuss the AC response when only one electrode is connected to the LL. (iii) Thirdly we consider application of an arbitrary AC electric field along the sample and compute the dynamical response of the LL with arbitrary interface resistances. The discussion is then specialized to the case of a uniform electric field.

PACS. 73.23.-b Electronic transport in mesoscopic systems – 71.10.Pm Fermions in reduced dimensions(anyons, composite fermions, Luttinger liquid, etc.) – 72.30.+q High-frequency effects; plasma effects

1 Introduction

The Luttinger liquid (LL) is one of the best understood strongly correlated system and departs strikingly from the more familiar Landau Fermi liquid with features such as spin-charge separation and charge fractionalization [1]. Of interest is the exploration of Luttinger physics in a mesoscopic context: several materials at the mesoscopic scale such as the quantum wires or the carbon nanotubes have been indeed proposed as realizations of a Luttinger liquid [2]. In this regard AC transport probes are an important tool because they allow access to the non-equilibrium physics of the LL.

In this paper we discuss AC transport in a Luttinger liquid resistively connected to two reservoirs through arbitrary interface resistances (not necessarily quantized at half a quantum of resistance): we will consider in turn (i) a gated Luttinger liquid, (ii) the response when one disconnects one of the reservoirs and (iii) application of an arbitrary AC electric field along the sample.

The first calculation of the dynamical conductivity was done for the inhomogeneous Luttinger liquid model (where one models the reservoirs as 1D non-interacting Fermi systems) by Safi [3]; the conductance for settings (i) and (iii) was later recovered by Ponomarenko [3]. The results for the gated wire were again later recovered using RPA by Blanter, Hekking and Büttiker who were the first to consider explicitly the gate conductance (i.e. the displacement current contribution) [4] and using the inhomogeneous LL model and then a boundary conditions formalism by Safi [5]; the case of a constant electric field was also considered by Sablikov and Shchamkhalova with results consistent with Ponomarenko's; appealing to Shockley's theorem they however claim that the real current measured at an electrode is not given by the electron current but add a displacement current contribution due to a charging of the reservoir caused by the charging of the wire itself: for a uniform electric field this results in a net current measured equal to a spatial average of the current through the wire [6]. Another approach has been advocated by Cuniberti, Sasseti and Kramer who consider an infinite system with long-range interactions and compute an absorptive conductance which has the advantage of being measurable by absorption of electromagnetic radiations without application of voltage or current probes [7]. An interesting development in several of these groups has been a focus on both capacitive and inductive effects with consideration of the kinetic inductance of a LL [7,8]. A transmission line approach to AC transport in a LL has also been proposed by Burke [9] to investigate the plasmon physics of LL based on earlier works on single-walled [10] and multi-walled carbon nanotubes [11]; the LL is modelled as a RL line coupled capacitively to a ground voltage. Additionally Burke discusses plasmon damping, a topic rather

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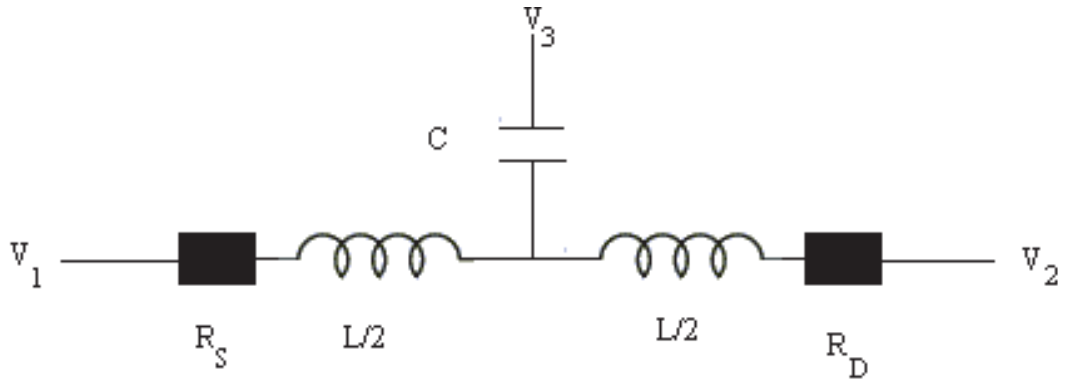

Fig. 1. Electrical circuit equivalent to the Luttinger liquid. The inductance per unit length is $\mathcal{L} = \frac{h}{2u\,Ke^2}$ and the capacitance per unit length is $\mathcal{C} = \frac{2Ke^2}{h\,u}$.

unexplored so far in that context. Inclusion of Coulomb interactions has also been considered in several papers [7,8].

In the case of short-range interactions (the pure LL) it is noteworthy that the DC limit of earlier calculations corresponds to contact resistances quantized at $R_0 = e^2/h$ the quantum of resistance (or equivalently to interface sample-electrode resistances $R_0/2$). We consider in this paper a more general situation by allowing for interface resistances distinct from $R_0/2$: while leaving open the experimental possibility that interface resistances are quantized at $R_0/2$, this permits dirty contacts to electrodes, which a priori is a not too unreasonable assumption.

We generalize earlier expressions for the dynamical conductance in situations (i) and (iii) above [3–5], and additionally compute the dynamical impedance. Case (ii) where only one electrode is connected is considered because it is a setting paradigmatic of time-dependent transport where the role of displacement currents is especially clear. For case (i) we show explicitly that the LL can be represented by an equivalent electrical circuit comprised of interface resistances R_S and R_D connected in series to an intrinsic inductance (per unit length) $\mathcal{L} = \frac{h}{2u\,Ke^2}$ (which is not purely kinetic but includes the effect of interactions), the whole being capacitively coupled to the ground through an intrinsic LL capacitance (per unit length) $\mathcal{C} = \frac{2Ke^2}{h\,u}$ (see Fig. 1). This shows explicitly from first principles the validity of the transmission line analogy considered by Burke [9] (our results however do not assume Galilean invariance which implies in turn the relation $v_F = u\,K$ for the Fermi velocity).

We show that AC measurements of both the dynamical gate conductance G_{33} and the impedance of the system up to order two in frequency allow full determination of the Luttinger liquid parameters u and K (the plasmon velocity and interaction strength) and of the interface resistances R_S and R_D. In particular the expected parallel law addition for the charge relaxation resistance is explicitly recovered as $R_q^{-1} = R_S^{-1} + R_D^{-1}$.

The paper is organized as follows: in Section 2, we introduce a boundary condition formalism which allows for the modelling of reservoirs resistively contacted to the Luttinger liquid. In Section 3 we discuss the gated Luttinger liquid computing both dynamical conductance and impedance matrices, as well as the LL connected to a single reservoir. In Section 4, we impose an AC electric field.

2 Voltage drops at interfaces: modelling contact resistances through boundary conditions

2.1 Chiral chemical potential operators

We consider the standard Luttinger Hamiltonian for a wire of length $L = 2a$.

$$H = \int_{-a}^{a} dx\, \frac{hu}{2K} \left(\rho_+^2 + \rho_-^2\right) + eV_3 \left(\rho_+ + \rho_-\right). \qquad (1)$$

V_3 is a gate potential which controls the Fermi level of the LL, ρ_+ and ρ_- are chiral particle densities which obey the relation $\rho_\pm(x,t) = \rho_\pm(x \mp ut)$. Their sum is just the total particle density $\rho - \rho_0$ while the electrical current is simply $i(x,t) = eu\left(\rho_+ - \rho_-\right)$.

Physically the chiral densities correspond to right and left-moving plasmons: they are therefore distinct from the right and left moving electron densities. They are used throughout the literature because they diagonalize the Luttinger Hamiltonian while the electron variables do not.

We now define the following operators:

$$\mu_\pm(x,t) = \frac{\delta H}{\delta \rho_\pm(x,t)}. \qquad (2)$$

Physically they correspond to (canonical) chemical potential operators: their average value yields the energy needed to add a particle locally at position x to the chiral density: $\rho_\pm \longrightarrow \rho_\pm + \delta(x)$. Similar operators have been introduced in Safi's boundary conditions formalism [5]: the main difference being that we consider chiral chemical potentials linked to the eigenmodes of the Luttinger liquid (the plasmons) while she defines chemical potentials related to the left or right moving (bare) electrons. Such chiral operators are much more convenient for calculations since they are directly related to the LL eigenmodes.

From their definition it follows that:

$$\mu_\pm(x,t) = \frac{hu}{K}\rho_\pm(x,t) + eV_3(t), \tag{3}$$

and therefore:

$$i(x,t) = K\frac{e}{h}\left(\mu_+(x,t) - \mu_-(x,t)\right). \tag{4}$$

It is convenient to redefine the chemical potentials by taking V_3 as reference:

$$\mu'_\pm(x,t) = \mu_\pm(x,t) - eV_3(t) \tag{5}$$

and using the fact that these shifted operators have a chiral time evolution:

$$\mu'_\pm(x,t) = \mu'_\pm(x \mp ut), \tag{6}$$

it follows immediately that:

$$\mu'_+(a,\omega) = \exp i\phi\ \mu'_+(-a,\omega), \tag{7a}$$

$$\mu'_-(a,\omega) = \exp -i\phi\ \mu'_-(-a,\omega), \tag{7b}$$

where we have defined a phase ϕ as:

$$\phi = \omega\frac{2a}{u}. \tag{8}$$

2.2 Interface resistances as boundary conditions

Up to now the Luttinger liquid is free standing. In real experimental settings coupling to probes is however unavoidable but in the absence of an exact solution of the problem of a mesoscopic LL wire coupled to many electrodes we decide to model the coupling to reservoirs through boundary conditions imposed on the otherwise free standing Luttinger liquid.

To enforce that we appeal to Sharvin-Imry contact resistance [12]: at the interface between a reservoir and a ballistic wire there is a voltage drop between the electrode voltage and the mean chemical potential within the wire; for a two-terminal geometry this in turn implies the existence of a contact resistance which can be viewed as the series addition of two interface resistances. For constrictions adiabatically connected to the reservoirs the contact resistance is quantized as $R_0 = h/e^2$. But in general it need not be; as shown by Büttiker incoherent transport through barriers can affect quantization [13].

For the LL we therefore make the hypothesis that as far as transport is concerned the resistive coupling to reservoirs can be modelled as:

$$i(-a,t) = \frac{1}{R_S}\left(V_S(t) - \frac{\mu_+(-a,t) + \mu_-(-a,t)}{2e}\right) \tag{9a}$$

$$i(a,t) = \frac{1}{R_D}\left(\frac{\mu_+(a,t) + \mu_-(a,t)}{2e} - V_D(t)\right) \tag{9b}$$

In the above equations we have considered two electrodes connected at the boundaries of the LL, the left electrode being a source at voltage $V_S(t)$ and the right electrode being a drain at voltage $V_D(t)$. Currents are oriented from left to right.

We stress that these relations are operator ones: we work therefore in the Heisenberg representation. For computation of noise properties it is indeed crucial that these relations are enforced at the operator level and not as average; knowledge of a current average is insufficient to specify fluctuation properties.

These relations extend an earlier formalism developed by the author and collaborators [14]: the main difference is that earlier we considered the chemical potentials as uniform (as is the case in a DC context) in a grand-canonical approach while here we work in a canonical setting with local potentials, which is more suitable to the AC context.

We note in passing that such relations can be derived explicitly in several exactly solvable models: for instance for the inhomogeneous Luttinger liquid with interface resistances $R_S = R_D = R_0/2$; for a chiral Luttinger liquid connected by a point contact to a Fermi liquid with $R = R_0/2$ or more generally for a reservoir which is a LL with LL parameter K_{res} also connected through a point contact to the sample, the interface resistance is $R = R_0/2K_{res}$. In the appendix it is shown how several earlier formalisms can be viewed as special cases of our formalism. We also refer the reader to our earlier work [14].

Simple though these relations may seem they permit to go beyond earlier AC results found by using for instance the inhomogeneous LL model as will next be shown.

3 Dynamical response of a gated wire

3.1 Dynamical impedance

We now consider time-dependent source and drain voltages $V_S(t) = V_1 \exp i\omega t$ and $V_D(t) = V_2 \exp i\omega t$ and a gate voltage $V_3(t) = V_3 \exp i\omega t$ and compute the dynamical impedance and conductance matrices of the LL. The currents at the boundaries of the system then acquire the same time dependence; we define currents as entering the system:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} i(-a,\omega) \\ -i(a,\omega) \end{pmatrix}. \tag{10}$$

To enforce current conservation there will in general be a displacement current corresponding to the charging of the sample. In that section we fix the currents at the boundaries as $i_1 = i_1^0 \exp i\omega t$ and $i_2 = i_2^0 \exp i\omega t$. Therefore the source and drain voltages $V_1(t) = V_1 \exp i\omega t$ and $V_2(t) = V_2 \exp i\omega t$ can be viewed as responses to the currents.

The boundary conditions are therefore rewritten as:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{R_S}\left(V_1 - \frac{\mu_+(-a) + \mu_-(-a)}{2e}\right) \\ \frac{1}{R_D}\left(V_2 - \frac{\mu_+(-a) + \mu_-(-a)}{2e}\right) \end{pmatrix}. \tag{11}$$

(We work now only with Fourier components at frequency ω; to avoid cumbersome notations the frequency dependence for $\mu_\pm(\pm a,\omega)$ will be however omitted in the rest of the paper.) Using equations (4) and (7) it follows that:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \frac{Ke}{h} \begin{pmatrix} 1 & -e^{i\phi} \\ -e^{i\phi} & 1 \end{pmatrix} \begin{pmatrix} \mu'_+(-a) \\ \mu'_-(a) \end{pmatrix}. \tag{12}$$

Defining the vector $\overrightarrow{\mu}$ as:

$$\overrightarrow{\mu} = \begin{pmatrix} \mu'_+(-a) \\ \mu'_-(a) \end{pmatrix}, \tag{13}$$

and using equations (5, 7, 11) there follows:

$$\begin{aligned} \begin{pmatrix} V_1 - V_3 \\ V_2 - V_3 \end{pmatrix} &= \begin{pmatrix} \dfrac{\mu'_+(-a) + \mu'_-(-a)}{2e} \\ \dfrac{\mu'_+(-a) + \mu'_-(-a)}{2e} \end{pmatrix} + \begin{pmatrix} R_S & 0 \\ 0 & R_D \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \\ &= \frac{1}{e} \begin{pmatrix} \frac{1}{2} & \frac{1}{2}e^{i\phi} \\ \frac{1}{2}e^{i\phi} & \frac{1}{2} \end{pmatrix} \overrightarrow{\mu} \\ &\quad + \begin{pmatrix} R_S & 0 \\ 0 & R_D \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}. \end{aligned} \tag{14}$$

Inserting equation (12), there comes:

$$\begin{pmatrix} V_1 - V_3 \\ V_2 - V_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + K\overline{R_S} & e^{i\phi}\left(\frac{1}{2} - K\overline{R_S}\right) \\ e^{i\phi}\left(\frac{1}{2} - K\overline{R_D}\right) & \frac{1}{2} + K\overline{R_D} \end{pmatrix} \frac{\overrightarrow{\mu}}{e} \tag{15}$$

where

$$\overline{R_D} = R_D/R_0, \tag{16a}$$

$$\overline{R_S} = R_S/R_0 \tag{16b}$$

are normalized resistances measured against the quantum of resistance $R_0 = \dfrac{h}{e^2}$. Inverting now equation (12) and inserting it in equation (15) one gets:

$$\begin{aligned} \begin{pmatrix} V_1 - V_3 \\ V_2 - V_3 \end{pmatrix} &= \frac{h}{Ke^2} \\ &\quad \times \begin{pmatrix} \frac{1}{2} + K\overline{R_S} & e^{i\phi}\left(\frac{1}{2} - K\overline{R_S}\right) \\ e^{i\phi}\left(\frac{1}{2} - K\overline{R_D}\right) & \frac{1}{2} + K\overline{R_D} \end{pmatrix} \\ &\quad \times \begin{pmatrix} 1 & -e^{i\phi} \\ -e^{i\phi} & 1 \end{pmatrix}^{-1} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}. \end{aligned} \tag{17}$$

Defining the dynamical impedance matrix as:

$$\begin{pmatrix} V_1 - V_3 \\ V_2 - V_3 \end{pmatrix} = \underline{Z} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}, \tag{18}$$

one finally finds:

$$\underline{Z} = \begin{pmatrix} R_S + i\dfrac{R_0}{2K}\cot\phi & i\dfrac{R_0}{2K\sin\phi} \\ i\dfrac{R_0}{2K\sin\phi} & R_D + i\dfrac{R_0}{2K}\cot\phi \end{pmatrix}. \tag{19}$$

where $\phi = \omega\frac{L}{u}$ (L is the length of the system, and u is the plasmon velocity). This is the main result of this section.

3.2 Intrinsic inductance of the Luttinger liquid

We now consider the following experimental arrangement in order to measure the impedance of the LL:

$$i_1 = -i_2 = i_0 \exp i\omega t. \tag{20}$$

The impedance of the system is therefore related to the matrix elements of the full impedance matrix by:

$$Z = \frac{V_1 - V_2}{i_1} = \underline{Z}_{11} + \underline{Z}_{22} - \underline{Z}_{12} - \underline{Z}_{21} \tag{21}$$

and therefore:

$$Z = R_S + R_D - i\frac{R_0}{K}\tan\left(\frac{\phi}{2}\right). \tag{22}$$

That especially simple formula admits as low frequency limit:

$$Z = R_S + R_D - i\frac{R_0}{K}\frac{\omega L}{2u} + i\frac{R_0}{3K}\left(\frac{\omega L}{2u}\right)^3 + \mathcal{O}(\omega^3) \tag{23}$$

where $L = 2a$ is the size of the system.

Comments:

(i) This shows firstly that the total contact resistance results as it should be from a series addition of the two interface resistances R_S and R_D.

(ii) Secondly, since $Z = R_S + R_D - i\omega\,(\mathcal{L}L) + \mathcal{O}(\omega)$ there appears an inductance per unit length:

$$\mathcal{L} = \frac{h}{2u\ Ke^2}. \tag{24}$$

This is as it should be; indeed direct inspection of the Luttinger Hamiltonian shows that the Luttinger liquid must have an inductance precisely set at that value. Indeed:

$$H = \int_{-a}^{a} dx\ \frac{hu}{4K}\rho^2 + \frac{hu\,K}{4}\overline{j}^2$$

where $\overline{j} = \rho^0_+ - \rho^0_-$ is the difference between bare right and left electron densities (at right and left Fermi points $\pm k_F$).

Rewriting the Hamiltonian in terms of charge density and current:

$$\rho_e = e\ \rho;\ j_e = e\ u\, K \overline{j}$$

(the last expression follows from charge conservation and the equations of motion) there follows:

$$H = \int_{-a}^{a} dx\ \frac{hu}{4Ke^2}\rho_e^2 + \frac{h}{4u\, Ke^2} j_e^2$$

This shows that the LL has indeed an inductance per unit length $\mathcal{L} = \frac{h}{2u\ Ke^2}$ while the zero mode of the first term yields $\frac{hu}{4Ke^2 L}Q^2$ which shows there is a capacitance per unit length:

$$\mathcal{C} = \frac{2Ke^2}{h\, u}. \tag{25}$$

While there has been wide emphasis on the intrinsic capacitance of the Luttinger liquid [15,4] the fact that the LL possesses an intrinsic inductance is less well stressed: see however [7,10]. We note in passing that the term $\frac{h}{4u\ Ke^2} j_e^2$ in the Hamiltonian results from both kinetic energy and interactions: the inductance $\mathcal{L} = \frac{h}{2u\ Ke^2}$ has therefore a mixed origin and is not merely contrarily to what Burke argues a kinetic inductance [9]: this point is somewhat obscured by the fact that in Galilean invariant systems $v_F = u\, K$ which implies then that the intrinsic LL inductance assumes exactly the same value as in a non-interacting system; however since Galilean invariance is in general not realized the previous identity does not hold and a renormalization by the interactions of the kinetic inductance should follow. At any rate experiments can provide independent measurements of both u and K: therefore one need not assume that $v_F = u\, K$, since the validity or non-validity of that relation can be checked.

(iii) For carbon nanotubes assuming a length $L \sim 1\ \mu m$ and a plasmon velocity of the order of $v_F = 10^5\ ms^{-1}$ means that each successive term in the low-frequency expansion of the impedance goes as $R_0 \left(\frac{\omega}{100\mathrm{GHz}}\right)^n$. This implies that at already a frequency of about 10 kHz the inductive correction is $\delta Z/Z = 10^{-7}$. While the first order correction is quite measurable the next order (three) correction is much less accessible unless one goes to the GHz range.

3.3 Dynamical conductance

We now fix voltages; therefore the relation $i_1 = -i_2$ is not valid any more. As amply stressed by Büttiker there is a displacement current i_3 due to the charging of the system [16]. Current conservation is enforced only if one considers that additional current.

Inversion of the relation $\begin{pmatrix} V_1 - V_3 \\ V_2 - V_3 \end{pmatrix} = \underline{Z} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$ yields only the upper 2×2 part of the dynamical conductance matrix.

But using current conservation $\sum_k i_k = 0$ and gauge invariance which implies $\sum_j G_{ij} = 0 = \sum_i G_{ij}$ the matrix elements of the third column and the third line follow immediately. The full conductance matrix is then:

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \underline{G} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

with matrix elements:

$$\underline{G}_{11} = \frac{KG_0 \left[\frac{1}{2} + K\overline{R_D} + \exp i2\phi \left(\frac{1}{2} - K\overline{R_D}\right)\right]}{\Delta} \tag{26a}$$

$$\underline{G}_{12} = \underline{G}_{21} = \frac{-KG_0 \exp i\phi}{\Delta} \tag{26b}$$

$$\underline{G}_{22} = \frac{KG_0 \left[\frac{1}{2} + K\overline{R_S} + \exp i2\phi \left(\frac{1}{2} - K\overline{R_S}\right)\right]}{\Delta} \tag{26c}$$

$$\underline{G}_{13} = \underline{G}_{31} = -\underline{G}_{11} - \underline{G}_{12} \tag{26d}$$

$$\underline{G}_{23} = \underline{G}_{32} = -\underline{G}_{22} - \underline{G}_{21} \tag{26e}$$

$$\begin{aligned} \underline{G}_{33} &= \underline{G}_{11} + \underline{G}_{22} - \underline{G}_{12} - \underline{G}_{21} \\ &= \frac{KG_0}{\Delta} \times [1 + K\left(\overline{R_S} + \overline{R_D}\right) - \\ &\quad -2e^{i\phi} + e^{i2\phi}\left(1 - K\left(\overline{R_S} + \overline{R_D}\right)\right)] \end{aligned} \tag{26f}$$

where one has defined

$$\begin{aligned} \Delta = \left(\frac{1}{2} + K\overline{R_S}\right)\left(\frac{1}{2} + K\overline{R_D}\right) - e^{i2\phi} \\ \times \left(\frac{1}{2} - K\overline{R_S}\right)\left(\frac{1}{2} - K\overline{R_D}\right). \end{aligned} \tag{27}$$

A useful check is to set the interface resistances to $R_S = R_D = R_0/2$: one then recovers equations (10–11) of Blanter et al. [4]. Our formalism is therefore fully consistent with earlier results.

We now expand the gate conductance $\underline{G}_{33}$:

$$\begin{aligned} \underline{G}_{33} = &-i\mathcal{C}L\omega + \omega^2 (\mathcal{C}L)^2 R_q \\ &+i\omega^3 (\mathcal{C}L)^3 R_q^2 \left(1 + \frac{R_0^2}{4K^2 R_q R_C} - \frac{R_0^2}{12K^2 R_q^2}\right) \\ &+\mathcal{O}(\omega^3) \end{aligned} \tag{28}$$

where $\mathcal{C} = \frac{2Ke^2}{h\, u}$, $R_q = (R_S R_D)/(R_S + R_D)$ and $R_C = R_S + R_D$ is the contact resistance.

3.4 Discussion

The previous expression shows:

(i) firstly that the capacitance per unit length $\mathcal{C} = \frac{2Ke^2}{h\, u}$ is independent of the coupling to the reservoirs: this is quite sensible; its value is just that expected from a direct inspection of the Luttinger Hamiltonian (see above). Measuring both $\mathcal{L} = \frac{h}{2u\ Ke^2}$ and $\mathcal{C} = \frac{2Ke^2}{h\, u}$ therefore provides a direct way to get the values of u and K. As already noticed by Burke using the telegraphist equation [9] the plasmon velocity is just $\sqrt{\mathcal{L}\mathcal{C}} = \frac{1}{u}$ an identity well-known to electrical engineers while the transmission line impedance is just: $Z_0 = \sqrt{\mathcal{L}/\mathcal{C}} = \frac{1}{2K}$.

This justifies a posteriori the transmission line analogy proposed by Burke.

(ii) That capacitance $\mathcal{C}$ is fully electrochemical: in a general experimental setting as stressed by Büttiker [16] the full capacitance must take into account a geometrical capacitance describing the coupling to a wire and a chemical (density of states) contribution so that the total capacitance is $(L\mathcal{C})^{-1} = C_{chem}^{-1} + C_{geom}^{-1}$.

(iii) There appears a charge relaxation resistance R_q which obeys a parallel addition law: $R_q^{-1} = R_S^{-1} + R_D^{-1}$. This is quite sensible because relaxation probabilities should add for independent relaxation processes. The charge relaxation resistance is distinct from the contact resistance in that it corresponds to an RC time for the discharging of a system and not (directly) to energy dissipation [16]. We also observe that by measuring both impedance and gate conductance up to order two in frequency one can directly measure both interface resistances: *there is therefore no need to assume that they are a priori set at* $R_S = R_D = R_0/2$ *since this can be checked experimentally.*

(iv) Several approaches have been advocated for determining the Luttinger parameters using AC measurements. Ponomarenko [3], Sablikov and Shchamkhalova [6] proposed to measure the period of the conductance since they are oscillating functions of the parameter $\phi = \omega\frac{L}{u}$. This has been criticized by Blanter et al. who argue that the frequency is quite high (GHz range) [4]. Safi proposed to measure the conductance at low frequency and measure its deviation to the DC limit [5], by showing that at low frequency for a *s*ymmetric electric field arrangement one can neglect the displacement current so that $G_{12} = -G_{11} = G_0(1 + i\omega\frac{L}{2u\,K})$: this allows access to the product $u\,K$. Blanter et al. argued that such deviations are hard to identify and proposed to measure the gate conductance up to order three [4] to determine the values of u and K.

As is apparent from our discussion of the inductance of the Luttinger liquid a joint measurement of both impedance and gate conductance circumvents the need to go to order three in frequency: it is sufficient to go to order one, which means that measurements at the kHz range should be enough rather than the 100 GHz range. Our measurement of the impedance $Z = G_{33}/[G_{11}G_{33} - G_{13}^2]$ has the advantage over that of G_{11} proposed by Safi that fixing the currents as in an impedance measurement avoids the complications of displacement current. In addition note that our derivation does not require a symmetric configuration for the electrodes.

(v) But if conversely one is able to make measurements up to order three (i.e. up to $G_0\left(\frac{\omega}{100\ \mathrm{GHz}}\right)^3$, or at GHz range), the order third term instead of providing a *mere fit* of the LL theory to experiments now constitutes a distinct non-trivial prediction of the theory. As is obvious from equation (28) the simpler expression given by Blanter et al. $i\omega^3\left(\mathcal{C}L\right)^3 R_q^2\left(1 - \frac{1}{3K^2}\right)$, is invalid for unquantized interface resistances: although correct for quantized interface resistances it does not interpolate to arbitrary resistances. Charge relaxation and contact resistances enter in an intricate way. The same comment applies to the dynamical impedance:

$$Z = R_S + R_D - i\omega L\mathcal{L} + i\omega^3 L^3\frac{\mathcal{L}^2\mathcal{C}}{12} + \mathcal{O}(\omega^3).$$

This shows the need in experiments to use our more general expressions which make no a priori assumption on the values of the interface resistances. (vi) Most of the literature neglects the inductive aspects of the LL. However without taking them into account it is impossible to interpret in terms of a circuit representation the gate conductance corrections appearing in the third-order term: for a pure RC circuit $\underline{G}_{33} = -i\mathcal{C}L\omega + \omega^2\left(\mathcal{C}L\right)^2 R_q + +i\omega^3\left(\mathcal{C}L\right)^3 R_q^2$. The appearance of K^2 in the formulas (e.g. Eq. (28)) is indeed the direct translation of inductive corrections since $\mathcal{L}/\mathcal{C} = \frac{R_0^2}{4K^2}$.

(vii) Resonances.

(vii-a) From the conductance matrix expression one extracts resonance peaks (cancellation of the common denominator Δ in Eq. (27)) as:

$$\omega_n = n\frac{\pi u}{L} - i\frac{u}{2L}\ln\left[\frac{\left(\frac{1}{2} + K\overline{R_S}\right)\left(\frac{1}{2} + K\overline{R_D}\right)}{\left(\frac{1}{2} - K\overline{R_S}\right)\left(\frac{1}{2} - K\overline{R_D}\right)}\right]$$

$$= n\omega_1 - i\frac{u}{2L}\ln\left[\frac{\left(Z_0 + R_S\right)\left(Z_0 + R_D\right)}{\left(Z_0 - R_S\right)\left(Z_0 - R_D\right)}\right].$$

where we have introduced the characteristic impedance of the LL $Z_0 = \sqrt{\mathcal{L}/\mathcal{C}} = R_0/2K$.

The frequencies correspond of course to harmonics of the fundamental mode ω_1 of the cavity formed by the LL. These standing waves are however damped by the interface resistances.

Blanter et al noticed in (Ref. [4]) that for $K = 1$ these plasmons are infinitely damped; however that conclusion is only valid for $R_S = R_D = R_0/2$ as can be seen from our more general expression.

Actually we find a more general and very straightforward condition for the vanishing of these modes:

$$R_S = Z_0 \quad or \quad R_D = Z_0.$$

The standing waves disappear if there is an impedance matching of the interface resistances with the characteristic impedance of the LL. This is as it should be: as is well known there are no reflected waves in an LC transmission line when the line is terminated with a load impedance equal to its characteristic impedance.

(vii-b) The expressions found in (Ref. [4]) for the height of the resonances are further generalized as follows. For $\underline{G}_{11}$ and other matrix elements in the upper left 2×2 block the height of resonance peaks is independent of K and is $1/(R_S + R_D)$ the contact conductance. For the gate conductance $\underline{G}_{33}$ the resonance peaks only occur for odd harmonics and their height is $4/(R_S + R_D)$ instead of $4/R_0$.

(viii) Implications for noise measurements.

Shot noise provides information on the charge of the carriers in the LL. While it is predicted [17] that the backscattering current off an impurity in a LL should exhibit a shot noise $S_{I_B} = 2KeI_B$ in an infinite system it has been shown that in the inhomogeneous LL the presence of the leads washes out the signature of the interactions [18] so that $S_{I_B} = 2eI_B$.

The previous discussion (vii-a) shows an easy way out: in order to observe the fractional charge Ke it suffices to make an *i*mpedance matching of the system so that due to the absence of reflected plasmons, the LL effectively responds as in an infinite system.

There are many ways to do that: (i) on the mesoscopic level for quantum wires one way would be to use gates close to the ends of the wire so that the coupling to the 2D electron gas can be tuned resulting in varying interface resistances; this is more difficult to do with carbon nanotubes. (ii) Another way is to macroscopically tune the total load resistances at source and drain: in our work the values of the resistances enter as parameters which model the reservoirs. These numbers make up the environment in our modelization. Since the contacts are already macroscopic we expect the results found to be quite robust even with such a macroscopic impedance matching.

How do we know that impedance matching has been achieved? The answer is easy: the conductance resonance peaks should be completely washed out (since there are no standing waves any more when impedances are matched).

3.5 AC response of a LL connected to a single reservoir

That experimental setting is especially interesting because in the DC limit there is no current. The dynamics within an AC experiment is wholly governed by the charge dynamics within the sample and illustrates nicely the role of the displacement current as stressed by Büttiker [16].

To the author's knowledge such a setting for a LL has not been treated in the literature even in the case of $R_S = R_0/2$. Yet in our formalism that situation is quite straightforwardly described: it suffices to take the limit of infinite interface resistance R_D if one wants for instance to disconnect the drain electrode. No current can flow into the drain and the current i_2 is therefore zero. The only non-zero matrix elements of the dynamical conductance are $\underline{G}_{11}, \underline{G}_{13}, \underline{G}_{33}, \underline{G}_{31}$ and are determined by a single number:

$$\underline{G}_{11} = \underline{G}_{33} = \frac{KG_0\left(1-\exp i2\phi\right)}{\left(\frac{1}{2}+K\overline{R_S}\right)+\exp i2\phi\left(\frac{1}{2}-K\overline{R_S}\right)}, \quad (29a)$$

$$\underline{G}_{13} = \underline{G}_{31} = -\underline{G}_{11}. \quad (29b)$$

In that situation the incoming current charges the Luttinger liquid and therefore $i_1 + i_3 = 0$: in other words the displacement current compensates exactly the charge current. The impedance $Z = \frac{V_1 - V_3}{i_1}$ is the inverse of $\underline{G}_{11}$:

$$\begin{aligned} Z &= R_S + \frac{i}{2K}\cot\phi \\ &= \frac{1}{-i\mathcal{C}L\omega} + R_S + \mathcal{O}(1). \end{aligned} \quad (30)$$

The low-frequency expansion of the conductance is:

$$\begin{aligned} \underline{G}_{33} = &-i\mathcal{C}L\omega + \omega^2\left(\mathcal{C}L\right)^2 R_S \\ &+ i\omega^3\left(\mathcal{C}L\right)^3 R_S^2\left(1-\frac{R_0^2}{12K^2R_S^2}\right). \end{aligned} \quad (31)$$

In such a setting the inductive effects are much more difficult to see: in both the conductance and the impedance they appear as $\left(\frac{\omega}{GHz}\right)^2$ corrections to the leading term in contrast with the two-terminal setting where for the impedance the inductive effects appear already at order $\left(\frac{\omega}{GHz}\right)$. Such a setting is however interesting in that it allows evidently to extract independently the values of each of the interface resistances.

4 Dynamical response of a LL to an AC electric field

4.1 Equations of motion and boundary conditions

We now apply an AC electric field along the sample and give therefore a spatial dependence to V_3:

$$V_3(x,t) = V_3(x)\, e^{i\omega t}. \quad (32)$$

The Luttinger Hamiltonian can be rewritten in terms of the phase field θ conjugate to the density as (we set $\hbar = 1$):

$$H = \int_{-a}^{a} dx\ \frac{\pi u}{2K}\rho^2 + \frac{u\,K}{2\pi}\left(\partial_x\theta\right)^2 + eV_3\rho \quad (33)$$

The equations of motion in the Heisenberg representation for the density and the particle current are:

$$\partial_t^2\rho - u^2\partial_x^2\rho = \frac{u\,Ke}{\pi}\partial_x^2 V_3 \quad (34a)$$

$$\partial_t^2 j - u^2\partial_x^2 j = -\frac{u\,Ke}{\pi}\partial_{x,t}^2 V_3 \quad (34b)$$

where the particle current is (as can be checked from the current conservation equation):

$$j = \frac{-u\,K}{\pi}\partial_x\theta.$$

These operators can therefore be written as:

$$\rho(x,t) = \rho_+(t-\frac{x}{u}) + \rho_-(t+\frac{x}{u}) + \rho_0(x)e^{i\omega t} \quad (35a)$$

$$j(x,t) = j_+(t-\frac{x}{u}) + j_-(t+\frac{x}{u}) + j_0(x)\, e^{i\omega t} \quad (35b)$$

where $\rho_0(x)e^{i\omega t}$ and $j_0(x)\, e^{i\omega t}$ are arbitrary particular solutions of the equations of motion. One can choose $\rho_0(x)$

to be proportional to $j_0(x)$. Indeed using current conservation $\partial_t\rho + \partial_x j = 0$ it follows immediately that:

$$\begin{aligned}\partial_x j &= -\partial_t\rho = -\partial_t\left(\rho_+ + \rho_-\right) - i\omega\rho_0(x)e^{i\omega t} \\ &= u\partial_x\left(\rho_+ - \rho_-\right) - i\omega\rho_0(x)e^{i\omega t} \end{aligned} \tag{36}$$

which implies that we can set:

$$j_\pm = \pm u\rho_\pm, \qquad \rho_0 = \frac{-1}{i\omega}\partial_x j_0. \tag{37}$$

The chiral chemical potential operators are now:

$$\mu_\pm(x,t) = \frac{2\pi u}{K}\rho_\pm(x,t) + eV_3(x,t) + \frac{\pi u}{K}\rho_0 \pm \frac{\pi}{K}j_0. \tag{38}$$

It is readily checked that the current operator is given by equation (4):

$$i(x,t) = ej(x,t) = K\frac{e}{h}\left(\mu_+(x,t) - \mu_-(x,t)\right). \tag{39}$$

Again it is convenient to shift the chemical potential operators to have operators which have a purely chiral time-evolution:

$$\mu'_\pm(x,t) = \mu_\pm(x,t) - eV_3(t) - \frac{\pi u}{K}\rho_0 \mp \frac{\pi}{K}j_0. \tag{40}$$

We now consider the following boundary conditions:

$$ej(-a) = \frac{1}{R_S}\left(V_3(-a) - \frac{\mu_+(-a,t) + \mu_-(-a)}{2e}\right), \tag{41a}$$

$$ej(a) = \frac{1}{R_D}\left(\frac{\mu_+(a) + \mu_-(a)}{2e} - V_3(a)\right). \tag{41b}$$

These boundary conditions correspond to source and drain voltages set to the ground (zero voltage): therefore only the potential V_3 appears; it corresponds to the energy gained due to the initial (or final) acceleration given by the applied electric field. In the previous section one did not have to take it into account since no electric field was applied.

4.2 Dynamical response

Defining again the currents as entering the system:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} ej(-a) \\ -ej(a) \end{pmatrix}, \tag{42}$$

the boundary conditions are rewritten as:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{R_S}\left(-\frac{\pi u}{eK}\rho_0(-a) - \frac{\mu'_+(-a) + \mu'_-(-a)}{2e}\right) \\ \frac{1}{R_D}\left(\frac{\pi u}{eK}\rho_0(a) - \frac{\mu'_+(-a) + \mu'_-(-a)}{2e}\right) \end{pmatrix}. \tag{43}$$

Therefore:

$$\begin{aligned}\begin{pmatrix} -\frac{\pi u}{eK}\rho_0(-a) \\ \frac{\pi u}{eK}\rho_0(a) \end{pmatrix} &= \begin{pmatrix} \frac{\mu'_+(-a) + \mu'_-(-a)}{2e} \\ \frac{\mu'_+(-a) + \mu'_-(-a)}{2e} \end{pmatrix} \\ &\quad + \begin{pmatrix} R_S & 0 \\ 0 & R_D \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \qquad (44) \\ &= \frac{1}{e}\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\exp i\phi \\ \frac{1}{2}\exp i\phi & \frac{1}{2} \end{pmatrix}\overrightarrow{\mu} \\ &\quad + \begin{pmatrix} R_S & 0 \\ 0 & R_D \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \qquad (45)\end{aligned}$$

where the vector $\overrightarrow{\mu}$ is defined as above (Eq. (13)). But according to equation (4) the current is rewritten as:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \frac{Ke}{h}\begin{pmatrix} 1 & -e^{i\phi} \\ -e^{i\phi} & 1 \end{pmatrix}\overrightarrow{\mu} + \begin{pmatrix} ej_0(-a) \\ -ej_0(a) \end{pmatrix}. \tag{46}$$

Therefore substitution of the previous equation in equation (44) yields:

$$\begin{aligned}\begin{pmatrix} -\frac{\pi u}{eK}\rho_0(-a) \\ \frac{\pi u}{eK}\rho_0(a) \end{pmatrix} &= \begin{pmatrix} \frac{1}{2} + K\overline{R_S} & e^{i\phi}\left(\frac{1}{2} - K\overline{R_S}\right) \\ e^{i\phi}\left(\frac{1}{2} - K\overline{R_D}\right) & \frac{1}{2} + K\overline{R_D} \end{pmatrix} \\ &\quad \times \frac{\overrightarrow{\mu}}{e} + \begin{pmatrix} ej_0(-a)R_S \\ -ej_0(a)R_D \end{pmatrix}. \qquad (47)\end{aligned}$$

Elimination of $\overrightarrow{\mu}$ then yields:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \underline{Z}^{-1}\begin{pmatrix} -\frac{\pi u}{eK}\rho_0(-a) - ej_0(-a)R_S \\ \frac{\pi u}{eK}\rho_0(a) + ej_0(a)R_D \end{pmatrix} + \begin{pmatrix} ej_0(-a) \\ -ej_0(a) \end{pmatrix} \tag{48}$$

where the matrix $\underline{Z}$ is the same dynamical impedance matrix found above in equation (19):

$$\underline{Z} = \begin{pmatrix} R_S + i\frac{R_0}{2K}\cot\phi & i\frac{R_0}{2K\sin\phi} \\ i\frac{R_0}{2K\sin\phi} & R_D + i\frac{R_0}{2K}\cot\phi \end{pmatrix}. \tag{49}$$

$\underline{Z}^{-1}$ is just the 2×2 upper restriction of the dynamical conductance matrix $\underline{G}$ found for the gated LL. Equation (48) is the main result of this sub-section. It can be rewritten as:

$$\begin{aligned}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} &= \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \underline{Z}^{-1}\begin{pmatrix} R_S + i\frac{1}{\omega\mathcal{C}}\partial_x & 0 \\ 0 & R_D + i\frac{1}{\omega\mathcal{C}}\partial_x \end{pmatrix}\right] \\ &\quad \times \begin{pmatrix} ej_0(-a) \\ -ej_0(a) \end{pmatrix} \qquad (50)\end{aligned}$$

where $\mathcal{C} = \frac{2Ke^2}{h\,u}$ is the intrinsic LL capacitance and where for instance

$$j_0(x) = \frac{i\omega eK}{u\pi}\int_0^x dy\int_0^y dz E(z)e^{i\frac{\omega}{u}(x+z-2y)}. \tag{51}$$

It is easily checked that the current response is independent of the particular solution j_0 chosen: shifting j_0 and $\rho_0 = \frac{-1}{i\omega}\partial_x j_0$ by either chiral currents δj_+ or δj_- so that $j_0' = j_0 + \delta j_+ + \delta j_-$ and a similar shift for the density ρ_0' gives contributions to equation (50) which cancel each other.

Even for the subcase $R_S = R_D = R_0/2$ this matrix equation does not seem to appear in the literature: for instance Ponomarenko writes the current at a given position in the inhomogeneous LL model for an arbitrary electric field as a Green function convolution [3] but that simple matrix relation between the response of a gated wire and the response to an arbitrary electric field is not explicitly written. Joint measurements in both contexts would be interesting to reveal such relations between the dynamical responses.

4.3 Uniform electric field

We now specialize the discussion to a uniform electric field $E = -\partial x V_3$ and choose the particular solution:

$$j_0' = -\frac{i\,u\,K e}{\omega\,\pi}E = -i2G_0\frac{u}{\omega}E\,.$$

Note that the particular solution j_0' chosen here is not the limiting case of equation (51) with a uniform electric field. Here the algebra is simpler with such a uniform current j_0': but of course as explained above the results do not depend on the choice made for j_0. The current response is therefore:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = ej_0'\left[\underline{Z}^{-1}\begin{pmatrix} R_S \\ -R_D \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right] = -iE\frac{G_0 u\,K}{\omega\Delta} \times \begin{pmatrix} \left(\frac{1}{2}+K\overline{R_D}\right) - \left(\frac{1}{2}-K\overline{R_D}\right)e^{i2\phi} - 2K\overline{R_D}e^{i\phi} \\ -\left[\left(\frac{1}{2}+K\overline{R_S}\right) - \left(\frac{1}{2}-K\overline{R_S}\right)e^{i2\phi} - 2K\overline{R_S}e^{i\phi}\right] \end{pmatrix}. \quad (52)$$

where

$$\Delta = \left(\frac{1}{2}+K\overline{R_S}\right)\left(\frac{1}{2}+K\overline{R_D}\right) - e^{i2\phi} \times \left(\frac{1}{2}-K\overline{R_S}\right)\left(\frac{1}{2}-K\overline{R_D}\right). \quad (53)$$

If $R_S = R_D = R_0/2$, this yields:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = -iE\frac{2G_0 u\,K}{\omega}\frac{-i\sin\frac{\phi}{2}}{K\cos\frac{\phi}{2} - i\sin\frac{\phi}{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad (54)$$

which is exactly the expressions found by Sablikov et al. [6] and Ponomarenko [3]. Sablikov and Shchamkhalova argue that due to a charging of the reservoirs the real current measured in an AC experiment is not i_1 but that one must add a displacement current $\frac{dQ_S}{dt}$ where Q_S is the charge appearing at the source [6,8]. Appealing to a result initially derived by Shockley, using Laplace equation they find that:

$$\frac{dQ_S}{dt} = -i_1 + \frac{1}{L}\int_{-L/2}^{L/2} i(x)\,dx,$$

and therefore the current measured at the left electrode is:

$$i_{mes} = \frac{1}{L}\int_{-L/2}^{L/2} i(x)\,dx,$$

for *a uniform electric field* and *plane electrodes orthogonal to the wire*.

That point of view is however valid only if one does not take into account relaxation processes in the reservoir: the charging of the reservoir must be taken into account only for frequencies $\omega \gg 1/\tau_{rel}$ where τ_{rel} is the relaxation time of the reservoir, i.e. the inverse of the plasma frequency $\omega_P = 1/\tau_{rel} \sim 10^{15}$ Hz. For optical processes this becomes relevant but not for the transport experiments one considers here.

It is quite easy to extract the distribution of current and charge in the sample:

$$i(x,\omega) = K\frac{e}{h}\left(\mu_+'(x,\omega) - \mu_-'(x,\omega)\right) + ej_0(x). \quad (55)$$

Since:

$$\mu_+'(x,\omega) = \exp i\frac{\phi}{2}\exp i\frac{\omega x}{u}\;\mu_+'(-a,\omega), \quad (56a)$$

$$\mu_-'(x,\omega) = \exp i\frac{\phi}{2}\exp -i\frac{\omega x}{u}\;\mu_-'(a,\omega), \quad (56b)$$

it follows:

$$i(x) = K\frac{e}{h}\exp i\frac{\phi}{2}\left(\exp i\tfrac{\omega x}{u}, -\exp -i\tfrac{\omega x}{u}\right)\cdot\overrightarrow{\mu} + ej_0(x). \quad (57)$$

Using the relation between $\overrightarrow{\mu}$ and j_0 (Eq. (47) above where one takes $\rho_0 = 0$ because the electric field is uniform) one easily finds:

$$i(x) = ej_0 - \frac{ej_0 K}{\Delta} \times \left[\cos(\frac{\omega x}{u})\left(\overline{R_S}+\overline{R_D}\right)\left(\cos\frac{\phi}{2} - i4K\frac{R_q}{R_0}\sin\frac{\phi}{2}\right) - \sin(\frac{\omega x}{u})\sin\frac{\phi}{2}\left(\overline{R_S}-\overline{R_D}\right)\right]. \quad (58)$$

For the symmetric case $R_S = R_D = R_0/2$ this reduces to:

$$i(x) = ej_0\left[1 - \frac{K\cos(\frac{\omega x}{u})}{K\cos\frac{\phi}{2} - i\sin\frac{\phi}{2}}\right],$$

which is also found by Sablikov et al. [6].

The density is then easily found as $\rho(x) = -\frac{1}{i\omega}\partial_x i(x)$.

5 Conclusions

In this paper we have discussed consequences on AC transport of the inclusion of arbitrary interface resistances R_S and R_D between the sample and the source and drain electrodes. The resistive coupling of the Luttinger liquid to the electrodes is described using a boundary condition formalism.

We considered a gated two-port Luttinger liquid which enabled us to generalize expressions of the dynamical conductance matrix. By considering the dynamical impedance we were able in particular to show that in the low-frequency limit the Luttinger liquid can be modelled as an electrical circuit comprising an inductance per unit length $\mathcal{L} = \frac{h}{2u\ Ke^2}$ in series with the interface resistances, the whole being capacitively coupled to the ground with intrinsic conductance per unit length $\mathcal{C} = \frac{2Ke^2}{h\,u}$.

We wish to stress the relevance of a study of a gated Luttinger liquid ohmically contacted to two reservoirs with *arbitrary* interface resistances: this is indeed the building block allowing for the description of more complicated circuits with ohmic or capacitive contacts with the LL. To give just one example: consider a LL connected to two gates (one facing the left end of the LL on a length l_1 and the second one facing the LL on a length l_2 at the right end, while a length l in the middle of the LL is left alone). If apart from these two gates, the LL is capacitively insulated from the ground (by being deposited for instance over an insulating substrate), then the impedance of the whole setup follows immediately: it is the series sum of three impedances, (i) one corresponding to a length l_1 gated LL connected with zero interface resistance to one reservoir ($R_S = \infty$, $R_D = 0$), (ii) the impedance of a free LL on a length l (found by taking $R_S = 0 = R_D$) and (iii) the impedance of a length l_2 gated LL connected with zero interface resistance to one reservoir ($R_D = \infty$, $R_S = 0$). The calculation of such an impedance would have been impossible with other formalisms such as the radiative boundary conditions [19] or the inhomogeneous LL.

Focusing in the impedance response of the LL we have shown that a joint measurement of both dynamical impedance and gate conductance G_{33} up to order one in frequency is sufficient to extract the Luttinger parameters. A measurement up to order two allows extraction of the interface resistances whose quantization can therefore be checked (or disproved); we also showed how they can be extracted from the setup of a gated LL connected to a single electrode. We found that the conductance resonances corresponding to standing waves in the LL are washed out if impedance matching is realized, i.e. if the interface resistances are such that $R_S = Z_0 = R_D$ where $Z_0 = \sqrt{\mathcal{L}/\mathcal{C}} = R_0/2K$ is the characteristic impedance of the LL. This again confirms the validity of viewing the LL as a quantum LC line. We then considered the application of an arbitrary AC electric field along the sample; we finally discussed the case of a uniform electric field generalizing earlier results valid only for $R_S = R_D = R_0/2$.

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Appendix A

We discuss in this appendix the relation of our formalism with several earlier approaches showing that they all imply our boundary conditions specialized to $R_S = R_D = \frac{h}{2e^2}$.

A.1 Radiative boundary conditions (RBC)

The basis of the RBC due to Egger and Grabert are the following two equations:

$$\rho^0_{R/L}(\mp a) = \frac{eV_{S/D}}{2\pi\hbar v_F}. \tag{59}$$

(These are Eqs. (3.1) and (3.2) of Ref. [19] where we have renamed the source and drain voltages and also the length to comply with our notations.)

$\rho^0_{R/L}$ are the bare *injected* electron densities (densities at $\pm k_F$ the right and left Fermi points). These equations just follow from noticing that the density of states is $\frac{1}{\pi\hbar v_F}$ and that a factor of $1/2$ must be added because only left or right moving electrons can be injected at either boundaries.

Using equations (3.3–3.6) of the same Reference these equations can be reset as the RBC i.e. equations (3.7–3.8):

$$\left(\frac{1}{K^2}\partial_x \pm \frac{1}{v_F}\partial_t\right)\langle\theta(x = \mp a, t\rangle = \frac{eV_{S/D}}{\sqrt{\pi}\hbar v_F}.$$

which are indeed radiative boundary conditions imposed on the field θ, hence the name of the method (note that Egger and Grabert take also a ground state average).

Equations (3.3–3.4) in reference [19] relate the bare densities to the true chiral densities (eigenmodes of the LL):

$$\rho^0_{R/L} = \frac{K^{-2} \pm 1}{2}\rho_+ + \frac{K^{-2} \mp 1}{2}\rho_-.$$

In terms of the chiral densities equation (59) above become:

$$\frac{K^{-2} \pm 1}{2}\rho_+(\mp a) + \frac{K^{-2} \mp 1}{2}\rho_-(\mp a) = \frac{eV_{S/D}}{2\pi\hbar v_F},$$

or:

$$\pm\frac{hv_F}{2e}\left[\rho_+(\mp a) - \rho_-(\mp a)\right] = V_{S/D} - \frac{hv_F K^{-2}}{2e}\left[\rho_+(\mp a) + \rho_-(\mp a)\right].$$

The authors also assume Galilean invariance with $v_F = u\ K$. Using the fact that $\mu_\pm = \frac{hu}{K}\rho_\pm$, there follows immediately $\frac{hv_F K^{-2}}{2}\left[\rho_+(\mp a) + \rho_-(\mp a)\right] = V_{S/D} - \frac{\mu_+ + \mu_-}{2e}$. Since the current is $I = u\ Ke\left[\rho_+ - \rho_-\right] = v_F\ e\left[\rho_+ - \rho_-\right]$ the equations obtain:

$$\pm\frac{h}{2e^2}I(\mp a) = V_{S/D} - \frac{\mu_+ + \mu_-}{2e}$$

which are exactly our boundary conditions specialized to the case of boundary interface resistances $R_S = R_D = \frac{h}{2e^2}$.

A.2 Blanter, Hekking, Büttiker's RPA

These authors assume also (Eq. (2) in Ref. [4]) that

$$\rho^0_{R/L}(\mp a) = \frac{eV_{S/D}}{2\pi\hbar v_F}$$

before making an RPA treatment. Therefore their approach is again equivalent to ours in the particular case $R_S = R_D = \frac{h}{2e^2}$.

A.3 Safi's bare electrons chemical potential boundary conditions

They are written as (Eq. (6) of the second reference in [5]):

$$\frac{\partial H}{\partial \rho^0_\pm} = eV_{S/D}$$

where $\rho^0_\pm$ are the bare electron densities within the wire.

Important remark: these densities differ from the densities $\rho^0_{R/L}$ above considered by Egger and Grabert because Egger and Grabert deal with electron densities injected *before screening* while the densities $\rho^0_\pm$ are the real injected densities for electrons at the right and left Fermi point *after screening* has been taken into account. Egger and Grabert find for instance that injection of the total unscreened density $\rho^0_R + \rho^0_L$ results in a real density variation $\rho = \rho_+ + \rho_- = \rho^0_+ + \rho^0_-$ within the wire equal to $\rho = K^2\left(\rho^0_L + \rho^0_L\right)$).

In summary there are three kind of densities: (i) *unscreened* densities of right and left moving electrons ρ^0_R and ρ^0_L; (ii) the real injected right and left moving *electron* densities ρ^0_+ and ρ^0_-; (iii) the real chiral densities ρ_+ and ρ_- within the LL which obey: $\rho_\pm(x,t) = \rho_\pm(x \mp ut)$ and which describe right and left moving *plasmons* in the LL (and NOT right and left moving electrons): these variable are used throughout this paper because they alone diagonalize the LL Hamiltonian while the right and left moving electron densities only diagonalize the non-interacting system.

The relation between $\rho^0_\pm$ and $\rho_\pm$ is the following one (see e.g. [14]):

$$\rho_\pm = \frac{1 \pm K}{2}\rho^0_+ + \frac{1 \mp K}{2}\rho^0_-.$$

It then follows immediately that:

$$\begin{aligned}\frac{\partial H}{\partial \rho^0_\pm} &= \frac{1 \pm K}{2}\frac{\partial H}{\partial \rho_\pm} + \frac{1 \mp K}{2}\frac{\partial H}{\partial \rho_\pm} \\ &= \frac{1 \pm K}{2}\mu_+ + \frac{1 \mp K}{2}\mu_-.\end{aligned}$$

Therefore:

$$\pm\frac{K}{2e}\left(\mu_+ - \mu_-\right) = V_{S/D} - \frac{\mu_+ + \mu_-}{2}$$

and since (i) $\mu_\pm = \frac{hu}{K}\rho_\pm$, (ii) $I = u\ Ke\left[\rho_+ - \rho_-\right]$ one recovers our boundary conditions again specialized to the case $R_S = R_D = \frac{h}{2e^2}$.

A.4 The inhomogeneous Luttinger liquid

The inhomogeneous Luttinger liquid [3] is a LL with spatially varying LL parameter $K(x)$ and $u(x)$ with $K(x) = K$ and $u(x) = u$ for $|x| \leq a$ and $K(x) = 1$, $u(x) = v_F$ for $|x| > a$. The central part is assumed to be the standard LL while the right and left parts are the leads modellized as 1D Fermi liquids. The Hamiltonian is written as:

$$H = \int_{-\infty}^{\infty} dx \frac{hu(x)}{4K(x)}\left(\rho^0_+ + \rho^0_-\right)^2 + \frac{hu(x)K(x)}{4}\left(\rho^0_+ - \rho^0_-\right)^2.$$

Charge conservation imposes that $\partial_t\rho + \partial_x I = 0$. In addition Hamilton equations of motion imply the equation $\partial_x\left(\frac{u(x)}{K(x)}\rho\right) + \frac{1}{u(x)K(x)}\partial_t I = 0$ (they can be found for example by writing ρ in terms of the standard phase field ϕ as $\frac{1}{\sqrt{\pi}}\partial_x\phi$ with a canonical conjugate field $\Pi = \sqrt{\pi}\left(\rho^0_+ - \rho^0_-\right)$ and $I = -\frac{1}{\sqrt{\pi}}\partial_t\phi$). Therefore both $\frac{u(x)}{K(x)}\rho$ and the current I must be continuous for $x = \pm a$. If we reexpress this in terms of the electron density this means that $\frac{u(x)}{K(x)}\left(\rho^0_+ + \rho^0_-\right)$ and $I(x) = u(x)K(x)\left(\rho^0_+ - \rho^0_-\right)$ are continuous. This implies

$$\begin{aligned}\frac{u}{K}\left(\rho^0_+ + \rho^0_-\right)|_{\pm a} &= v_F\left(\rho^0_+ + \rho^0_-\right)|_{leads}, \\ u\ K\left(\rho^0_+ - \rho^0_-\right)|_{\pm a} &= v_F\left(\rho^0_+ - \rho^0_-\right)|_{leads},\end{aligned}$$

and therefore

$$\frac{u}{K}\left[\rho^0_+(-a^+) + \rho^0_-(-a^+)\right] + u\ K\left[\rho^0_+(-a^+) + \rho^0_-(-a^+)\right] = 2v_F\ \rho^0_+(-a^-),$$

and:

$$\frac{u}{K}\left[\rho^0_+(a^-) + \rho^0_-(a^-)\right] - u\ K\left[\rho^0_+(a^-) + \rho^0_-(a^-)\right] = 2v_F\ \rho^0_-(a^+).$$

Using the fact that within the leads $\rho^0_\pm(\mp a) = \frac{eV_{S/D}}{hv_F}$ and $\frac{\partial H}{\partial \rho^0_\pm} = \frac{hu(x)}{2K(x)}\left(\rho^0_+ + \rho^0_-\right) \pm \frac{hu(x)K(x)}{2}\left(\rho^0_+ - \rho^0_-\right)$ there comes

$$\frac{\partial H}{\partial \rho^0_\pm} = eV_{S/D}.$$

This shows that the inhomogeneous LL implies approach (A.3) by Safi. The proof we have given above follows that given by Safi in reference [5]. In summary the inhomogeneous LL implies that that there are interface resistances at the boundaries of the LL which are quantized as: $R_S = R_D = \frac{h}{2e^2}$.

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